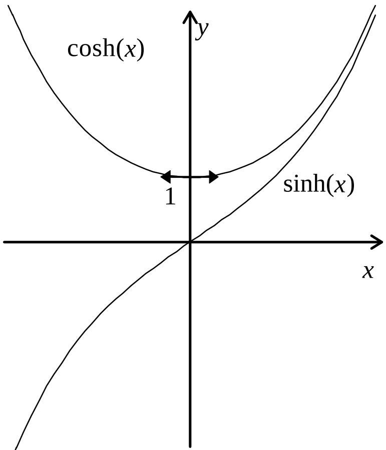
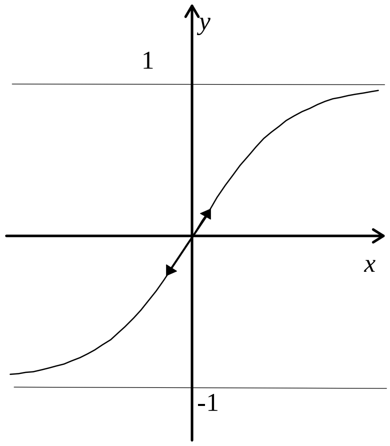
Formulaire

I Fonction hyperboliques

1°- Définitions



Fonctions cosh(x) et sinh(x)



Fonction tanh(x)

$$\cosh x = \frac{e^{ix} + e^{-x}}{2} \qquad \sinh x = \frac{e^{x} - e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1} = \frac{1 - e^{-2x}}{1 + e^{-2x}}$$

$$e^{x} = \cosh x + \sinh x \quad e^{-x} = \cosh x - \sinh x$$

$$\cosh^2 x - \sinh^2 x = 1$$

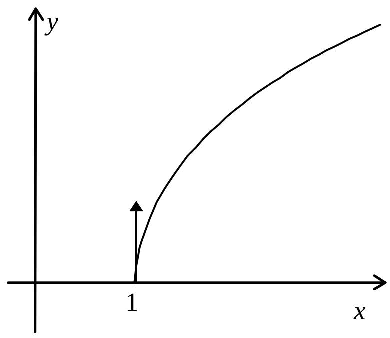
2°- Formule de Moivre et applications

$$(\cosh x + \sinh x)^n = \cosh nx + \sinh nx$$
$$(\cosh x - \sinh x)^n = \cosh nx - \sinh nx$$

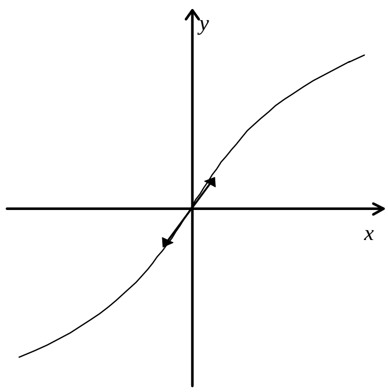
Appliquée avec la formule du binôme, on retrouve les formules de linéarisation.

Formule du binôme : $(a+b)^n = \sum_{p=0}^{n} C_n^p \, a^{n-p} \, b^p$

3°- Fonctions hyperboliques réciproques



Fonction argcosh(x)

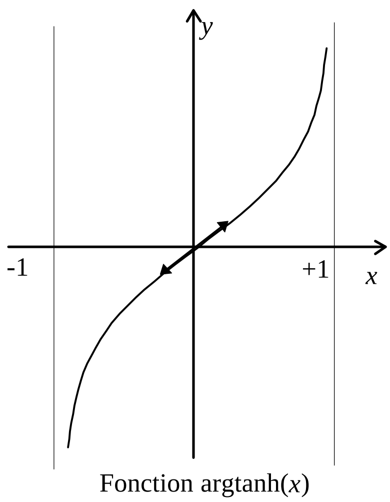


Fonction argsinh(x)

$$\arg\cosh x = \ln\left(x + \sqrt{x^2 - 1} \right)$$

$$\arg\sinh x = \ln\left(x + \sqrt{x^2 + 1} \right)$$

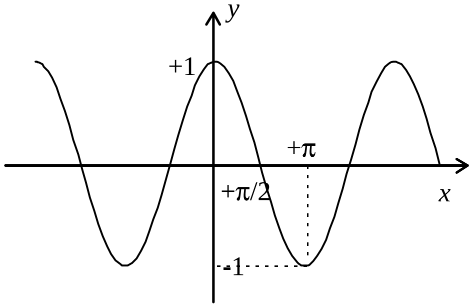
$$\arg\tanh x = \frac{1}{2} \ln \frac{1+x}{1-x}$$



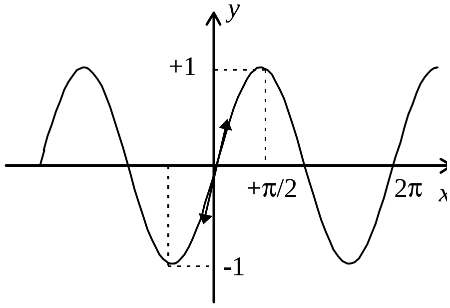
Fonction argtanh(x)

II Trigonométrie

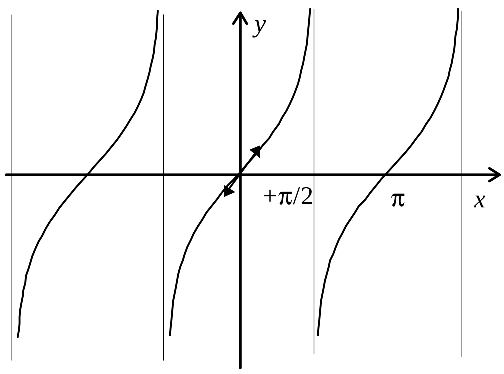
1°- Définitions



Fonction cos(x)



Fonction sin(x)



Fonction tan(x)

$$\cos x = \frac{e^{ix} + e^{-ix}}{2} \qquad \sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{1}{i}\frac{e^{ix} - e^{-ix}}{e^{ix} + e^{-ix}} = i\,\frac{e^{-ix} - e^{ix}}{e^{-ix} + e^{ix}} = i\,\frac{1 - e^{2ix}}{1 + e^{2ix}}$$

$$\cot x = \frac{\cos x}{\sin x} = i\,\frac{e^{ix} + e^{-ix}}{e^{ix} - e^{-ix}}$$

$$e^{ix} = \cos x + i\sin x \qquad e^{-ix} = \cos x - i\sin x$$

$$\cos^2 x + \sin^2 x = 1$$

2°- Formules d'addition

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$
$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$
$$\sin(a-b) = \sin a \cos b - \cos a \sin b$$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b} \qquad \tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

$$\cos a \cos b = \frac{1}{2}\left[\cos(a+b) + \cos(a-b)\right]$$

$$\sin a \sin b = \frac{1}{2}\left[\cos(a-b) + \cos(a+b)\right]$$

$$\sin a \cos b = \frac{1}{2}\left[\sin(a+b) + \sin(a-b)\right]$$

$$\cos p + \cos q = 2\cos\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right)$$

$$\cos p - \cos q = -2\sin\left(\frac{p+q}{2}\right)\sin\left(\frac{p-q}{2}\right)$$

$$\sin p + \sin q = 2\sin\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right)$$

$$\sin p - \sin q = 2\sin\left(\frac{p-q}{2}\right)\cos\left(\frac{p+q}{2}\right)$$

$$\tan p + \tan q = \frac{\sin(p+q)}{\cos p \cos q} \qquad \tan p - \tan q = \frac{\sin(p-q)}{\cos p \cos q}$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\sin 2x = 2\sin x \cos x$$

$$\cos^2 x = \frac{1+\cos 2x}{2} \qquad \sin^2 x = \frac{1-\cos 2x}{2}$$

On pose : $t = \tan\left(\frac{x}{2}\right)$, alors :

$$\cos x = \frac{1-t^2}{1+t^2} \qquad \sin x = \frac{2t}{1+t^2} \qquad \tan x = \frac{1+t^2}{1-t^2}$$

$$e^{ix} = \frac{1+it}{1-it} \qquad t = \tan\left(\frac{x}{2}\right) = \frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x}$$

$$\cos(3x) = 4\cos^3 x - 3\cos x \qquad \sin(3x) = 3\sin x - 4\sin^3 x$$

$$\tan(3x) = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$$

3°- Formule de Moivre et applications

$$(\cos x + i\sin x)^n = \cos nx + i\sin nx$$
$$(\cos x - i\sin x)^n = \cos nx - i\sin nx$$

Appliquée avec la formule du binôme, on retrouve les formules de linéarisation.

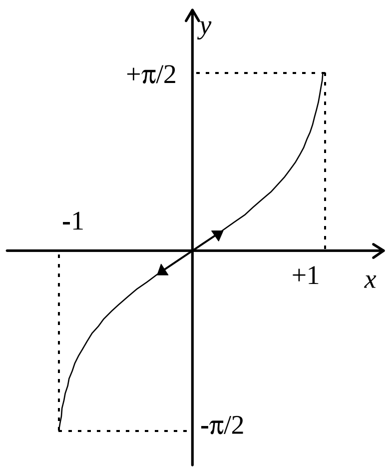
Formule du binôme : $(a+b)^n = \sum_{p=0}^{n} C_n^p \, a^{n-p} \, b^p$

4°- Fonctions circulaires réciproques

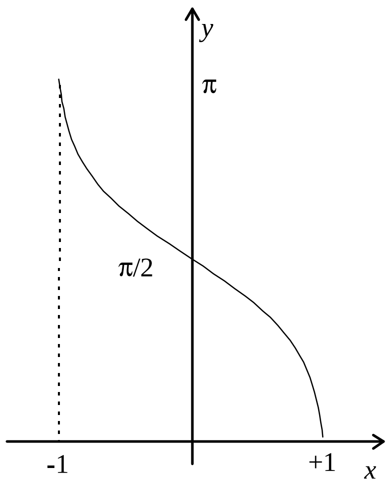
$$\arccos x + \arcsin x = \frac{p}{2}$$

$$\arctan x + \arctan \frac{1}{x} = \frac{p}{2} \quad si\ x > 0$$

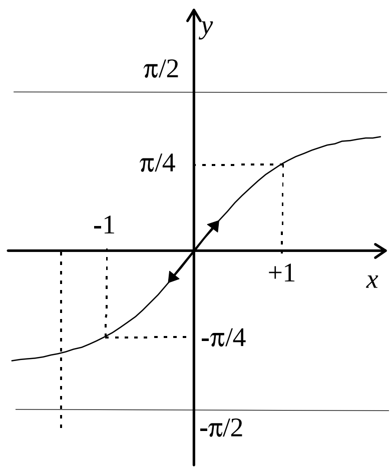
$$\arctan x + \arctan \frac{1}{x} = -\frac{p}{2} \quad si\ x < 0$$



Fonctions arcsin(*x*)



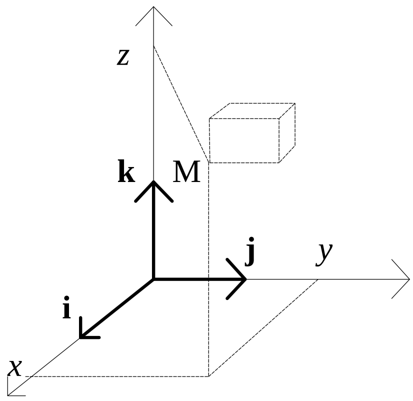
Fonction arccos(*x*)



Fonction arctan(*x*)

III Opérateurs différentiels

1°- Coordonnées cartésiennes



$$\mathbf{grad}U = \frac{\partial U}{\partial x}\mathbf{i} + \frac{\partial U}{\partial y}\mathbf{j} + \frac{\partial U}{\partial z}\mathbf{k}$$

$$\mathrm{div}\,\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

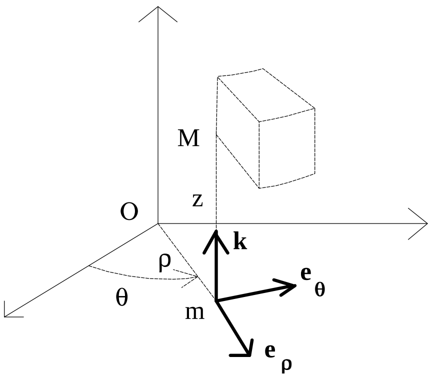
$$\mathbf{rot}\ \mathbf{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\mathbf{i} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\mathbf{j} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\mathbf{k}$$

$$\Delta U = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}$$

$$\Delta \mathbf{A} = \left(\Delta A_x\right)\mathbf{i} + \left(\Delta A_y\right)\mathbf{j} + \left(\Delta A_z\right)\mathbf{k}$$

Elément de volume: $d\tau = dx.dy.dz$

2°- Coordonnées cylindriques



$$\mathbf{grad}\, U = \frac{\partial U}{\partial \rho}\mathbf{e}_\rho + \frac{1}{\rho}\frac{\partial U}{\partial \theta}\mathbf{e}_\theta + \frac{\partial U}{\partial z}\mathbf{k}$$

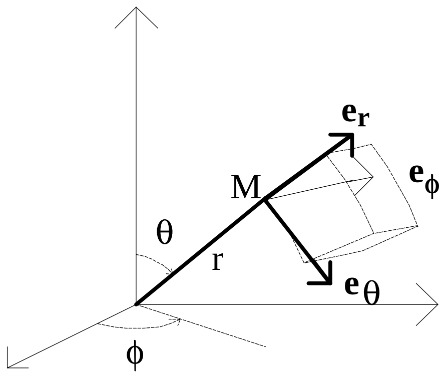
$$\mathrm{div}\,\mathbf{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho A_\rho\right) + \frac{1}{\rho}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$$

$$\mathbf{rot}\,\mathbf{A} = \left(\frac{1}{\rho}\frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z}\right)\mathbf{e}_\rho + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\mathbf{e}_\theta + \frac{1}{\rho}\left(\frac{\partial}{\partial \rho}\left(\rho A_\theta\right) - \frac{\partial A_\rho}{\partial \theta}\right)\mathbf{k}$$

$$\Delta U = \frac{\partial^2 U}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial U}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 U}{\partial \theta^2} + \frac{\partial^2 U}{\partial z^2}$$

Elément de volume: $d\tau = \rho d\rho . d\theta . dz$

3°- Coordonnées sphériques



$$\mathbf{grad}\, U = \frac{\partial U}{\partial r}\mathbf{e_r} + \frac{1}{r}\frac{\partial U}{\partial \theta}\mathbf{e}_\theta + \frac{1}{r\sin\theta}\frac{\partial U}{\partial \phi}\mathbf{e}_\phi$$

$$\mathrm{div}\,\mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta A_\theta\right) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\mathbf{rot}\ \mathbf{A} = \left(\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial \theta}\left(\sin\theta A_\phi\right) - \frac{\partial A_\theta}{\partial \phi}\right)\right)\mathbf{e_r} +$$

$$\frac{1}{\mathrm{r}}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r}\left(rA_\phi\right)\right)\mathbf{e}_\theta + \quad \frac{1}{\mathrm{r}}\left(\frac{\partial}{\partial r}\left(rA_\theta\right) - \frac{\partial A_r}{\partial \theta}\right)\mathbf{e}_\phi$$

$$\Delta U = \frac{\partial^2 U}{\partial r^2} + \frac{2}{r}\frac{\partial U}{\partial r} + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial U}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 U}{\partial \phi^2}$$

Elément de volume: $\mathrm{d}\tau = r^2\sin\theta\,\mathrm{d}r.\mathrm{d}\theta.\mathrm{d}\phi$

IV Formules d'analyse vectorielle

1°- Relations "de base"

$div(\mathbf{grad}\,U) = \Delta U$ $\quad\quad$ $\mathbf{rot}\,(\mathbf{grad}\,U) = \mathbf{0}$ $\quad\quad$ $\mathrm{div}\,(\mathbf{rotA}) = 0$

2°- Relations type "dérivée d'un produit"

$\mathbf{grad}(UV) = U\,\mathbf{grad}\,V + V\,\mathbf{grad}\,U$ $\quad\quad$ $\mathrm{div}(r\mathbf{A}) = r\,\mathrm{div}\mathbf{A} + \mathbf{grad}\,r\,.\,\mathbf{A}$

$$\mathbf{rot}(r\mathbf{A}) = r\,\mathbf{rot}\,\mathbf{A} + (\mathbf{grad}\,r)\times\mathbf{A}$$

Moins facile:

$\mathbf{rot}(\mathbf{rot}\,\mathbf{A}) = \mathbf{grad}\,(\mathrm{div}\mathbf{A}) - \Delta\mathbf{A}$ $\quad\quad$ $\mathrm{div}(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\,\mathbf{rot}\,\mathbf{A} - \mathbf{A}\,\mathbf{rot}\,\mathbf{B}$

$$\mathbf{rot}(\mathbf{A}\times\mathbf{B}) = \mathbf{A}\,\mathrm{div}\,\mathbf{B} - \mathbf{B}\,\mathrm{div}\,\mathbf{A} + (\mathbf{B.grad})\mathbf{A} - (\mathbf{A.grad})\mathbf{B}$$

V Relations intégrales

1°- Green-Ostrogradsky

$$\oiint_S \mathbf{AdS} = \iiint_V div\mathbf{A}\,dt$$

Volume V contenu dans la surface fermée S

Flux de **A** à travers S = intégrale de div **A** sur tout le volume V

2°- Stokes-Ampère

$$\oint_C \mathbf{A}\,\mathbf{dl} = \iint_S \mathbf{rot}\,\mathbf{A}\,\mathbf{dS}$$

Surface S appuyée sur le contour fermé C

Circulation de **A** sur C = flux de **rotA** à travers S

3°- Formules dérivées

S surface fermée entourant V :

$$\oiint_{S} U\,\mathbf{dS} = \iiint_{V} \mathbf{grad}U\,\mathrm{d}t$$

S surface appuyée sur le contour fermé orienté C :

$$\oint_{C} U\,\mathbf{dl} = -\iint_{S} \mathbf{grad}U \times \mathbf{dS}$$

S surface fermée entourant V :

$$\oiint_{S} \mathbf{A} \times \mathbf{dS} = -\iiint_{V} \mathbf{rotA}\,dt$$